**Fundamental Trigonometric Identities**

Identities enable us to simplify complicated expressions. They are the basic tools of trigonometry used in solving trigonometric equations, just as factoring, finding common denominators, and using special formulas are the basic tools of solving algebraic equations.

To verify trigonometric identities, we usually start with the more complicated side of the equation and essentially manipulate the expression until it has been transformed into the same expression as the other side of the equation. Sometimes we have to factor expressions, expand expressions, find common denominators, or use other algebraic strategies to obtain the desired result.

We recently manipulated the Pythagorean theorem

By dividing both sides by and using the definitions of sine and cosine to show

We then further manipulated it to show

and

This process of manipulating using algebra properties is at the heart of verifying identities.

Example 1: Verify

Example 2: Verify

Example 3: Create an identity by rewriting the expression in terms of

Example 4: Verify

Example 5: Verify

**Solving Trigonometric Equations by Factoring**

Occasionally we will encounter trigonometric equations that are quadratic in form. In these instances, we can still utilize factoring (or the quadratic formula) to assist in solving. In order to simplify the notation, some will use a quick substitution (often called substitution).

Example 6: Solve each equation on the interval

**Solving Trigonometric Equations with Identities**

When presented with an equation that has more than one trigonometric function, it is often best to manipulate the equation and use the ideas of trigonometric identities to first rewrite the equation using one function.

Example 7: Solve on the interval

Example 8: Solve on the interval

**Solving Trigonometric Equations with Multiple Angles**

Sometimes it is not possible to solve a trigonometric equation with identities that have a multiple angle, such or . When confronted with these equations, recall that is a horizontal compression by a factor of 2 of the function . Thus, on an interval of we could graph two periods of , as opposed to one cycle of . This compression of the graph leads us to believe there may be twice as many –intercepts or solutions to compared to .

Example 9: Solve on the interval